DERIVATION VECTORIELLE

Q1) $$\left(\frac{d\overrightarrow{AB}}{dt}\right)_{/B_1} = \left(\frac{da\,\vec{x}_2}{dt}\right)_{/B_1} = a\times\left(\frac{d\,\vec{x}_2}{dt}\right)_{/B_1} \Rightarrow \boxed{\left(\frac{d\overrightarrow{AB}}{dt}\right)_{/B_1} = +a\,\dot{\alpha}\,\vec{y}_2}$$

Q2) $$\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = \left(\frac{d\left(b\,\vec{x}_3\right)}{dt}\right)_{/B_1} = b\times\left(\frac{d\,\vec{x}_3}{dt}\right)_{/B_1} = b\times\frac{d}{dt}\left(\cos\beta\,\vec{x}_2+\sin\beta\,\vec{y}_2\right)_{/B_1}$$

$$\Rightarrow \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = b\times\left[\left(-\dot{\beta}\sin\beta\,\vec{x}_2+\cos\beta\,\dot{\alpha}\,\vec{y}_2\right)+\left(\dot{\beta}\cos\beta\,\vec{y}_2-\sin\beta\,\dot{\alpha}\,\vec{x}_2\right)\right]$$

$$\Rightarrow \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = b\times\begin{pmatrix}-\dot{\beta}\sin\beta-\dot{\alpha}\sin\beta\\ \cos\beta\,\dot{\alpha}+\dot{\beta}\cos\beta\\ 0\end{pmatrix}_{B_2} \Rightarrow \boxed{\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = b\times\left(\dot{\alpha}+\dot{\beta}\right)\times\begin{pmatrix}-\sin\beta\\ +\cos\beta\\ 0\end{pmatrix}_{B_2}}$$

$$\Rightarrow \boxed{\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = +b\times\left(\dot{\alpha}+\dot{\beta}\right)\times\vec{y}_3}$$

Q3) $$\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_2} + \overrightarrow{BC}\wedge\overrightarrow{\Omega_{1/2}} = b\,\dot{\beta}\,\vec{y}_3 + b\,\vec{x}_3\wedge -\dot{\alpha}\,\vec{z}_3$$

$$b\times\left(\frac{d\,\vec{x}_3}{dt}\right)_{/B_2} = b\times\dot{\beta}\times\vec{y}_3$$

$$-\overrightarrow{\Omega_{2/1}} = -\dot{\alpha}\,\vec{z}_i$$

$$\Rightarrow \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = b\,\dot{\beta}\,\vec{y}_3 + b\,\dot{\alpha}\,\vec{y}_3 \Rightarrow \boxed{\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = +b\times\left(\dot{\alpha}+\dot{\beta}\right)\times\vec{y}_3}$$

Q4) $$\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_3} + \overrightarrow{BC}\wedge\overrightarrow{\Omega_{1/3}} = \vec{0} + b\,\vec{x}_3 \wedge -(\dot{\alpha}+\dot{\beta})\vec{z}_3$$

$$b\times\left(\frac{d\vec{x}_3}{dt}\right)_{/B_3} = \vec{0}$$

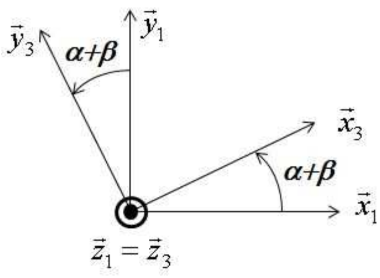
$$-\overrightarrow{\Omega_{3/1}} = -\left(\overrightarrow{\Omega_{3/2}}+\overrightarrow{\Omega_{2/1}}\right) = -\left(\dot{\beta}\,\vec{z}_i + \dot{\alpha}\,\vec{z}_i\right)$$

$$\Rightarrow \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = -b\times(\dot{\alpha}+\dot{\beta})\times\vec{x}_3\wedge\vec{z}_3 \Rightarrow \boxed{\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = +b\times(\dot{\alpha}+\dot{\beta})\times\vec{y}_3}$$

Q5) $$\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = b\times\left(\frac{d\vec{x}_3}{dt}\right)_{/B_1} = b\times\frac{d}{dt}\left[\cos(\alpha+\beta)\vec{x}_1 + \sin(\alpha+\beta)\vec{y}_2\right]_{/B_1}$$

$$\Rightarrow \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = b\times\left[-(\dot{\alpha}+\dot{\beta})\sin(\alpha+\beta)\,\vec{x}_1 + (\dot{\alpha}+\dot{\beta})\cos(\alpha+\beta)\,\vec{y}_1\right]$$

$$\Rightarrow \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = b\times(\dot{\alpha}+\dot{\beta})\times\overbrace{\left[-\sin(\alpha+\beta)\,\vec{x}_1 + \cos(\alpha+\beta)\,\vec{y}_1\right]}^{\vec{y}_3}$$



$$\Rightarrow \boxed{\left(\frac{d\overrightarrow{BC}}{dt}\right)_{/B_1} = +b\times(\dot{\alpha}+\dot{\beta})\times\vec{y}_3}$$