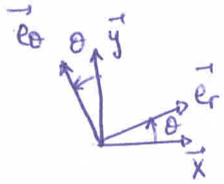
Frein à disque

Q 1) On isole le disque 2.

$$-N + pS = 0 \qquad S = \iint_{r_1,\,-\alpha}^{r_2,\,+\alpha} r\,dr\,d\theta = \frac{1}{2}\left(r_2^2 - r_1^2\right)\times 2\alpha = \left(r_2^2 - r_1^2\right)\alpha$$

donc $p = \dfrac{N}{(r_2^2 - r_1^2).\alpha}$

Q 2) $d\vec{F}_{2\to1} = -p\,dS.\vec{z} - p.f\,dS\,\vec{e_\theta}$ en M



modèle global : $\{\mathcal{F}_{2\to1}\} = \left\{ \begin{array}{l} \vec{R}_{2\to1} = \int_{(S)} d\vec{F}_{2\to1} \\ \vec{M}^{O}_{2\to1} = \int_{(S)} \vec{OM}\wedge d\vec{F}_{2\to1} \end{array} \right\}$

Q 3) $\vec{R}_{2\to1} = \int_{(S)} \left(-p\vec{z} - p.f.\vec{e_\theta}\right) dS$ avec $dS = r\,dr\,d\theta$ et $\vec{e_\theta} = \cos\theta\,\vec{y} - \sin\theta\,\vec{x}$

donc $\vec{R}_{2\to1} = -p\int_{(S)} dS\,\vec{z} - pf\int_{(S)}\cos\theta\,dS\,\vec{y} + \underbrace{pf\int_{(S)}\sin\theta\,dS\,\vec{x}}_{\vec{0}}$

$$\vec{R}_{2\to1} = -p.\left(r_2^2 - r_1^2\right)\alpha\vec{z} - pf.\cancel{2}\sin\alpha.\frac{r_2^2 - r_1^2}{\cancel{2}}\vec{y} + \vec{0}$$

$$\vec{R}_{2\to1} = -p\left(r_2^2 - r_1^2\right)\left(\alpha\vec{z} + f\sin\alpha\,\vec{y}\right)$$

$$\vec{M}^{O}_{2\to1} = \int_{(S)} \vec{OM}\wedge\left(-p\vec{z} - p.f\,\vec{e_\theta}\right)dS = -\int_{(S)}\left(r\vec{e_r} + e\vec{z}\right)\wedge\left(p\vec{z} + f.p.\vec{e_\theta}\right)dS$$

$$= -p\int\left(-r.\vec{e_\theta} + f r.\vec{z} - e f\,\vec{e_r}\right)dS$$

$$= -p\int\left[-r\left(\cos\theta\,\vec{y} - \sin\theta\,\vec{x}\right) + f.r.\vec{z} - e.f.\left(\cos\theta\,\vec{x} + \sin\theta\,\vec{y}\right)\right]dS$$

$$= \int_{(S)}\left[-p.r.\cancel{\sin\theta} + p.e.f\cos\theta\right]dS\,\vec{x} + \int_{(S)}\left[p.r.\cos\theta + \cancel{p.e.f\sin\theta}\right]dS\,\vec{y} - \int_{(S)} p.f.r\,dS\,\vec{z}$$

$$= p.e.f.\cancel{2}\sin\alpha.\frac{\left(r_2^2 - r_1^2\right)}{\cancel{2}}\vec{x} + p.2\sin\alpha\frac{\left(r_2^3 - r_1^3\right)}{3}\vec{y} - pf.2\alpha.\frac{\left(r_2^3 - r_1^3\right)}{3}\vec{z}$$

$C_f = -p.f.\frac{2}{3}.\alpha\left(r_2^3 - r_1^3\right)$ avec $p = \dfrac{N}{(r_2^2 - r_1^2)\alpha}$

donc $C_f = -\frac{2}{3}.f.N.\dfrac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$ (×2 car 2 plaquettes)